

THEOREM 4.3.1: For all positive integers a and b ,
if $a \mid b$, then $a \leq b$.

Proof: Let a and b be positive integers.

Suppose $a \mid b$. [N.T.S. $a \leq b$]

\therefore For some integer k , $b = ak$,
by definition of "divides".

Since $ak = b$, $k = \frac{b}{a}$ by Rule of Algebra.

Since $a > 0$ and $b > 0$, $k = \frac{b}{a} > 0$, by R.O.A.

Since $k > 0$ and k is an integer, $k \geq 1$.

So, $1 \leq k$.

$\therefore a \cdot 1 \leq ak$, by R.O.A.

Since $ak = b$ and $a \cdot 1 = a$, $a \leq b$, by substitution.

\therefore For all positive integers a and b ,

if $a \mid b$, then $a \leq b$, by Direct Proof.
Q.E.D.

Be able to cite Theorem 4.3.1 by Name, as in

"Since r and s are positive integers and $r \mid s$,
then $r \leq s$, by Theorem 4.3.1."